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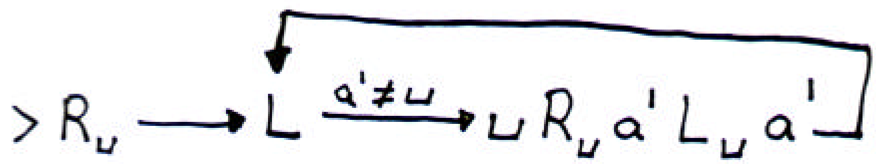
CSCIE 207
Homework Set #4

Problem #1 (4.2.1)



Problem #2 (4.3.4)
a) The operation of a pushdown automaton with two stacks (2PDA) is as follows:
Initially the machine is in the initial state s and the two stacks are empty. The transitions are of the form ((p,a,â1,â2), (q,ã1,ã2)). Whenever the machine is in state p with â1 at the top of the first stack and with â2 at the top of the second stack, it may read the symbol a from the input tape (if a=e, then the input is not consulted), replace â1 by ã1 on the top of the first stack, replace â2 by ã2 on the top of the second stack, and enter state q.
For the machine to be deterministic, there must be no two distinct compatible transitions, in other words, there can be no situation in which two different transitions are applicable. The machine accepts a string if it winds up in a final state after having consumed all input symbols and with both stacks empty. To decide a language, the 2PDA is constructed such that if it does not accept a string it rejects the string explicitly by going to a non-final “dead” state.
b) The class of recursive languages consists of the languages that are decided by Turing machines (TMs).
Now, the operation of a language deciding TM can be simulated by a 2PDA: The first stack is used to represent the contents of the tape of the TM to the right of the head, with the symbol at the top of the stack the symbol the head is currently positioned at. The second stack represents the symbols to the left of the head (excluding the beginning-of-tape symbol), with the symbol on top of the second stack being the symbol found one position to the left of the head.
To initialize the simulation, the 2PDA reads all symbols from its input, pushing them onto the second stack. Then it pops them one by one from the second stack, pushing them onto the first stack (pushing an extra blank at the end), thereby establishing the correct order of the symbols. Movement of the head of the TM to the right/left is simulated by popping one symbol from the first/second stack and pushing it onto the second/first stack. Writing a symbol to the tape of the TM is simulated by popping a symbol from the first stack and pushing the symbol to be written onto the first [1]stack. Transitions of the TM to an accepting configuration is simulated by going to a final

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state and popping all symbols from the two stacks (note that all input symbols have been consumed upon initialization of the simulation). Transition of the TM to a rejecting configuration is simulated by going to a non-final "dead" state.

On the other hand, any 2PDA can be simulated by a 2-tape TM which in turn, as we know, can be simulated by a standard TM. The first tape of the 2-tape TM represents the input to the 2PDA and its first stack (separated by a blank) and the second tape the second stack. Consumption of an input symbol by the 2PDA is simulated by overwriting that symbol on the first tape with a chosen special symbol that is not part of the input alphabet. Popping a symbol from one of the stacks is simulated by erasing the symbol at the right end of the corresponding tape. Pushing a symbol is simulated by writing the symbol at the position of the first blank at the right end of the tape. If it is detected that all input symbols have been consumed (overwritten), the two "stacks" are empty, and the corresponding 2PDA is in a final state, a transition is made to an accepting halting state. If the corresponding 2PDA transits to its special rejecting state, a transition is made to a rejecting halting state.

Thus 2PDAs are computationally equivalent to TMs and therefore decide precisely the class of recursive languages.

Problem #3 (5.4.1)

a) Consider a 2-tape TM named M' that performs on its first tape exactly the actions that M performs on its single tape. Upon initialization the second tape of M' is blank, however the k^{th} symbol on the second tape is not blank, it contains an arbitrary symbol, say a. Every time the head of the first tape moves to the right or left, the head of the second tape is moved in the same direction. If the second head is positioned on a non-blank symbol, (a), M' will halt. Thus, the problem is solvable.

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